Practice Midterm Exam

This exam is closed-book and closed-computer. You may have a double-sided, 8.5" × 11" sheet of notes with you when you take this exam. You may not have any other notes with you during the exam. You may not use any electronic devices (laptops, cell phones, etc.) during the course of this exam. Please write all of your solutions on this physical copy of the exam.

You are welcome to cite results from the problem sets or lectures on this exam. Just tell us what you're citing and where you're citing it from. However, please do not cite results that are beyond the scope of what we've covered in CS103.

> On the actual exam, there'd be space here for you to write your name and sign a statement saying you abide by the Honor Code. We're not collecting or grading this exam (though you're welcome to step outside and chat with us about it when you're done!) and this exam doesn't provide any extra credit, so we've opted to skip that boilerplate.

You have three hours to complete this practice midterm. There are 32 total points. This practice midterm is purely optional and will not directly impact your grade in CS103, but we hope that you find it to be a useful way to prepare for the exam. You may find it useful to read through all the questions to get a sense of what this practice midterm contains before you begin.

Question	Points
(1) Set Theory	/ 8
(2) Mathematical Logic	/ 8
(3) Proofwriting I	/ 8
(4) Proofwriting II	/ 8
	/ 32

Good luck!

Problem One: Set Theory   (8 Points)

(CS103 Midterm, Spring 2015)

Recall that if S and T are sets, the difference of S and T, denoted $S - T$, is defined as the set of all elements that are in S but not in T.

    i. **(5 Points)** Are there any sets A and B such that $\wp(A - B) = \wp(A) - \wp(B)$? If so, give an example of sets A and B with these properties. If not, prove why not.

(CS103 Midterm, Winter 2013)

ii. **(3 Points)** Let A and B be arbitrary sets and consider the set S defined below:

$$S = \{\, x \mid \neg(x \in A \to x \in B) \,\}$$

Write an expression for S in terms of A and B using the standard set operators (union, intersection, etc.), but without using set-builder notation. Briefly justify why your answer is correct.

Problem Two: Mathematical Logic   (8 Points)

(CS103 Midterm, Spring 2015)

Franz Kafka's *Before the Law* is an existentialist short story about a man who encounters an open gate guarded by a gatekeeper. The gatekeeper tells him that he should not enter, and over the years the man repeatedly tries and fails to persuade the gatekeeper to let him through. It is ultimately revealed that the man was the only person ever allowed to pass through this particular gate, though he never does so.

Before the Law only takes a few minutes to read, so if you haven't yet read it, I highly encourage you to do so after the exam. Right now, though, we'd like you to translate a summary of the story into first-order logic. It's existentialism meets existential quantifiers. ☺

i.   **(5 Points)** Given the predicates

Person(p), which states that p is a person;
Gate(g), which states that g is a gate;
MayPass(p, g), which states that p is permitted to pass through g; and
WillPass(p, g), which states that p will eventually pass through g,

write a statement in first-order logic that says "some person has a gate that they alone are permitted to pass through, but which they will never pass through."

(CS103 Midterm, Spring 2015)

On Problem Set Two, we asked you to translate the statement "every set has a power set" into first-order logic. Here is one possible answer:

$$\forall S.\ (Set(S) \rightarrow$$
$$\exists P.\ (Set(P) \wedge$$
$$\forall T.\ (T \in P \leftrightarrow Set(T) \wedge \forall x \in T.\ x \in S)$$
$$)$$
$$)$$

ii. **(3 Points)** Give a statement in first-order logic that is the negation of this statement. As in Problem Set Two, your final formula must not have any negations in it, except for direct negations of predicates.

Problem Three: Proofwriting I   (8 Points)

(CS103 Midterm, Fall 2016)

On Problem Set One, you explored Yablo's Paradox, a list of statements that make claims about one other. This problem explores a different list of mutually interacting statements, though (fortunately) this one doesn't involve any paradoxes.

Consider the following list of 137 statements:

(S_1): There is exactly 1 false statement in this list.
(S_2): There are exactly 2 false statements in this list.
(S_3): There are exactly 3 false statements in this list.
...
(S_{135}): There are exactly 135 false statements in this list.
(S_{136}): There are exactly 136 false statements in this list.
(S_{137}): There are exactly 137 false statements in this list.

More generally, statement (S_n) is the statement "There are exactly n false statements in this list."

Which of these statements are true, and which of these statements are false? Prove it. For full credit, you should prove both that

- your answer is correct, and

- no other possible answer is correct.

As a hint to help you get started, you might want to think about how many statements in the list can be true at the same time.

*(Extra space for your answer to Problem Three, if you need it.)*

Problem Four: Proofwriting II   (8 Points)

(CS103 Midterm, Fall 2016)

On Problem Set Two, you explored tournaments, contests between groups of *n* players. This problem explores some other properties of tournaments.

Let's begin by refreshing some definitions. A ***tournament*** is a contest among *n* players. Each player plays a game against each other player, and either wins or loses the game (let's assume that there are no draws).
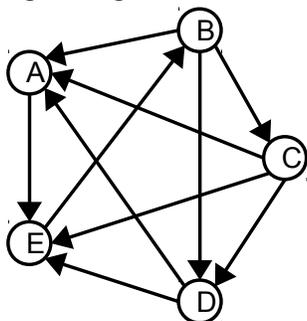
We can visually represent a tournament by drawing a circle for each player and drawing arrows between pairs of players to indicate who won each game. For example, in the tournament to the left, player *A* beat player *E*, but lost to players *B*, *C*, and *D*.

A ***tournament winner*** is a player in a tournament who, for each other player, either won her game against that player, or won a game against a player who in turn won his game against that player (or both). In the tournament to the left, players *B*, *C*, and *E* are tournament winners. However, player *D* is ***not*** a tournament winner, because he neither beat player *C*, nor beat anyone who in turn beat player *C*. Although player *D* won against player *E*, who in turn won against player *B*, who then won against player *C*, under our definition player *D* is ***not*** a tournament winner.

Now, here's a new definition. A ***tournament loser*** is a player in a tournament who, for each other player, either *lost* her game against that player, or lost a game against a player who in turn lost his game against that player (or both). For example, in the tournament shown above, players *A*, *B*, and *E* are tournament losers. Notice that players *B* and *E* are simultaneously tournament winners and tournament losers – these properties are not mutually exclusive! This problem explores an unusual interplay between these definitions.

Let *T* be an arbitrary tournament where everyone is a tournament loser. Prove that every player in *T* is also a tournament winner. (There's probably a good life lesson in here?)

(Extra space for your answer to Problem Four, if you need it.)